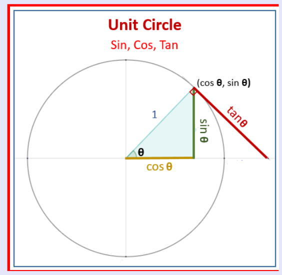
Trigonometry

Lecture 49



Feb 19-8:47 AM

Given $r = -6 \sin\theta$

1) Convert to rectangular coordinate system.

$$r \cdot r = -6\, r \sin\theta$$

$$r^2 = -6\, r\sin\theta$$

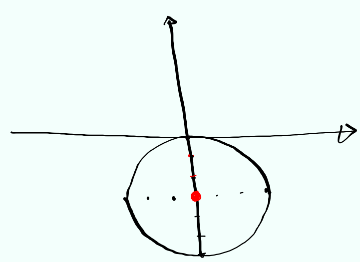
$$x^2 + y^2 = -6y$$

$$x^2 + y^2 + 6y = 0$$

2) Graph $\quad x^2 + y^2 + 6y + 9 = 9$

$$(x-0)^2 + (y+3)^2 = 3^2$$

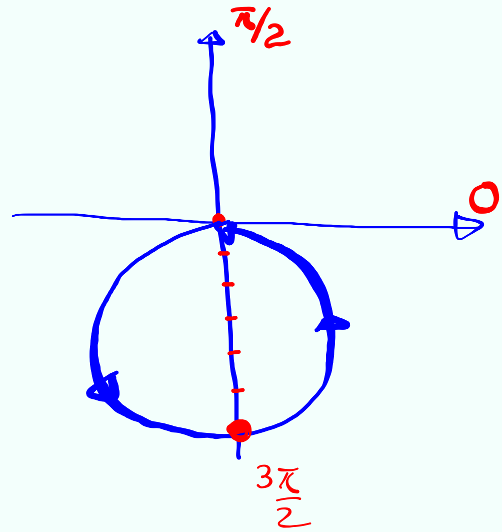
Circle center $(0,-3)$ Radius 3



Dec 2-10:29 AM

$r = -6\sin\theta$

θ	r
0	0
$\frac{\pi}{2}$	-6
π	0
$\frac{3\pi}{2}$	6
2π	0

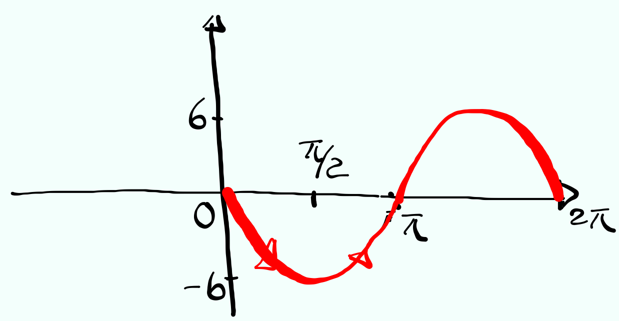


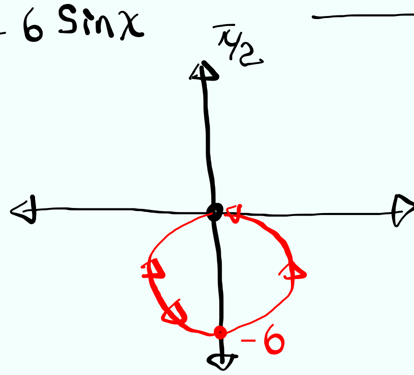
Dec 2-10:34 AM

$r = -6\sin\theta$

Hold!

$y = -6\sin x$

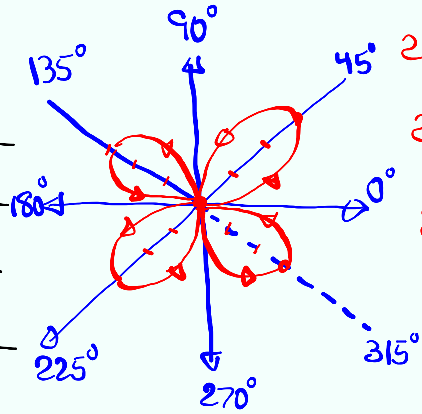




Dec 2-10:36 AM

Graph $r = 3 \sin 2\theta$

θ	r
0	0
45°	3
90°	0
135°	-3
180°	0



$2\theta = 0 \rightarrow \theta = 0$

$2\theta = 90^\circ \rightarrow \theta = 45^\circ$

$2\theta = 180^\circ \rightarrow \theta = 90^\circ$

$2\theta = 270^\circ \rightarrow \theta = 135^\circ$

$2\theta = 360^\circ \rightarrow \theta = 180^\circ$

$2\theta = 450^\circ \rightarrow \theta = 225^\circ$

Dec 2-10:40 AM

$r = 3 \sin 2\theta$

$0 \le \theta \le 2\pi$

$0 \le 2\theta \le 4\pi$

Hold!

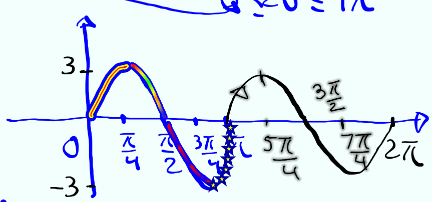
$y = 3 \sin 2x$

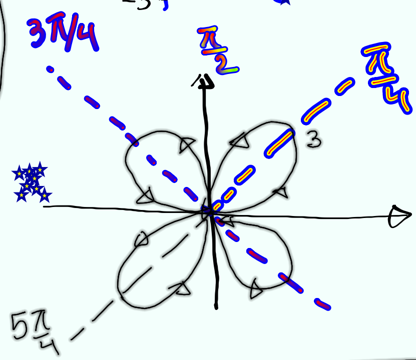
$0 \le 2x \le 2\pi$

$0 \le x \le \pi$

$n = 2$

4 petals





$r = a \sin n\theta$

$r = a \cos n\theta$

Rose

n is odd $\Rightarrow n$ petals

n is even $\rightarrow 2n$ petals

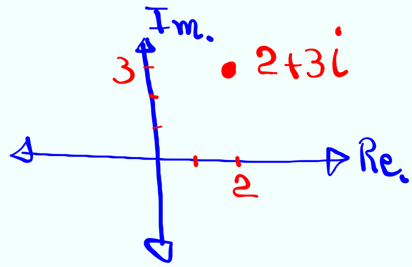
Dec 2-10:48 AM

Complex numbers

$$Z = a + bi \qquad i = \sqrt{-1} \qquad i^2 = -1$$

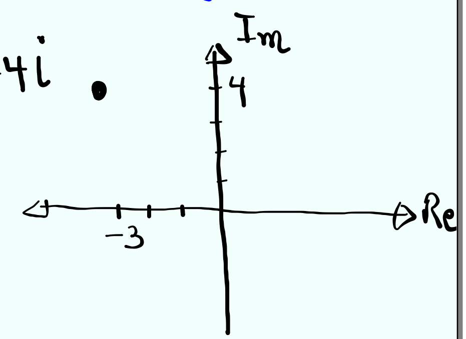
Real Part (a)

Imaginary Part (b)



$Z = 2 + 3i$

$Z = -3 + 4i$



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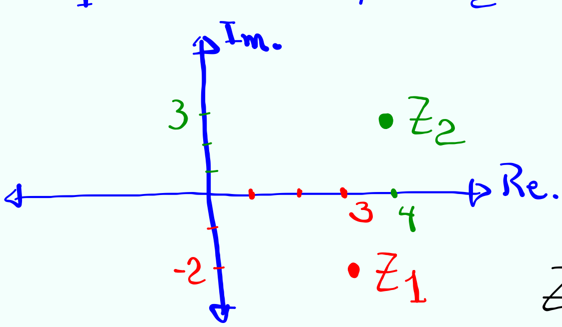
$Z_1 = 3 - 2i$, $Z_2 = 4 + 3i$



$$Z_1 + Z_2 = 3 - 2i + 4 + 3i$$

$$= \boxed{7 + i}$$

$$Z_1 - Z_2 = 3 - 2i - 4 - 3i$$

$$= -1 - 5i$$

Dec 2-11:01 AM

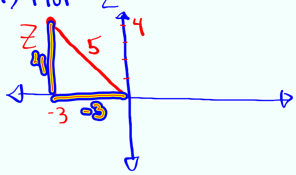
If $Z = a + bi$, then

$$|Z| = \sqrt{a^2 + b^2}$$

$$\underbrace{\text{Abs. value of } Z}_{\text{Modulus}}$$

Suppose $Z = -3 + 4i$

1) Plot Z



2) $|Z| = \sqrt{(-3)^2 + 4^2}$

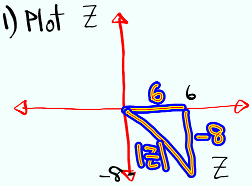
$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$

Suppose $Z = 6 - 8i$

1) Plot Z



2) $|Z| = \sqrt{6^2 + (-8)^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$

Dec 2-11:04 AM

Suppose $Z = 1 + \sqrt{3}\, i$

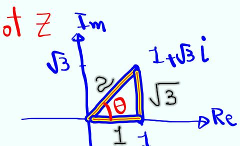
1) Real part

$a = 1$

2) Imaginary Part

$b = \sqrt{3}$

3) Plot Z



4) $|Z| = \sqrt{a^2 + b^2}$

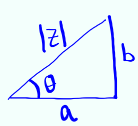
$= \sqrt{1^2 + (\sqrt{3})^2}$

$= \sqrt{1 + 3}$

$= \sqrt{4} = 2$

$\sin\theta = \frac{\sqrt{3}}{2}$, $\cos\theta = \frac{1}{2}$, $\tan\theta = \sqrt{3}$

$\theta = \tan^{-1}(\sqrt{3}) = 60° = \frac{\pi}{3}$



$\sin\theta = \frac{b}{|Z|} \rightarrow b = |Z| \sin\theta$

$\cos\theta = \frac{a}{|Z|} \rightarrow a = |Z| \cos\theta$

$a + bi = |Z| \cos\theta + |Z| \sin\theta\, i$

$= |Z| (\cos\theta + i \sin\theta)$

Polar or trig. form of complex #.

Dec 2-11:11 AM

$Z = 1 + \sqrt{3}\, i$

$|Z| = 2$

$\theta = \frac{\pi}{3}$

$\Rightarrow \boxed{2\left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right)}$

Dec 2-11:19 AM

$Z = -4\sqrt{3} - 4i$

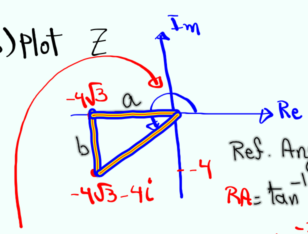
1) Real part

$a = -4\sqrt{3}$

2) Imaginary Part

$b = -4$

3) Plot Z



$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

Ref. Angle

$RA = \tan^{-1}\left(\frac{b}{a}\right)$

$= \tan^{-1}\left(\frac{4}{4\sqrt{3}}\right)$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$ or 30°

4) $|Z| = \sqrt{a^2 + b^2}$

$= \sqrt{(-4\sqrt{3})^2 + (-4)^2}$

$= \sqrt{16 \cdot 3 + 16}$

$= \sqrt{64} = 8$

$a + bi = |Z|\left(\cos\theta + i \sin\theta\right)$

$-4\sqrt{3} - 4i = \boxed{8\left(\cos\frac{7\pi}{6} + i \sin\frac{7\pi}{6}\right)}$

Complex form

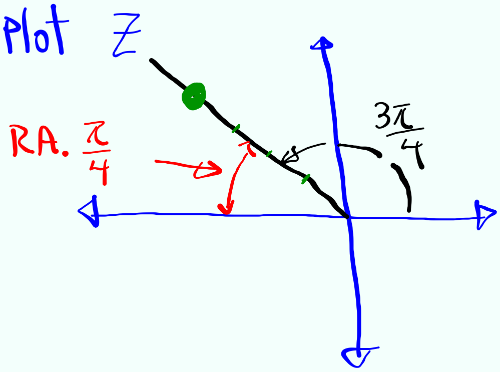
Polar form

Dec 2-11:21 AM

Given $Z = 4\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$r = |z|$ $\quad \theta = \frac{3\pi}{4}$

1) Plot Z



2) Find Z in $a+bi$ form

$$Z = 4\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$= 4\left(-\frac{\sqrt{2}}{2} + i\cdot\frac{\sqrt{2}}{2}\right)$$

$$= \boxed{-2\sqrt{2} + 2\sqrt{2}\,i}$$

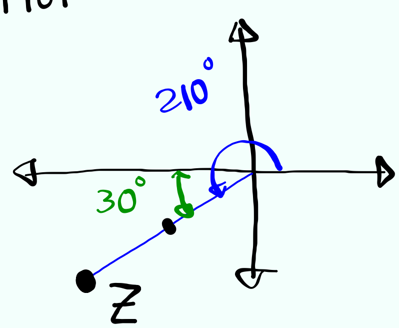
Dec 2-11:30 AM

Given $Z = 2\,\text{cis}\,210^\circ$

$$= 2\left(\cos 210^\circ + i\sin 210^\circ\right)$$

$|z| = r$

1) Plot



$$= 2\left(-\frac{\sqrt{3}}{2} + i\cdot -\frac{1}{2}\right)$$

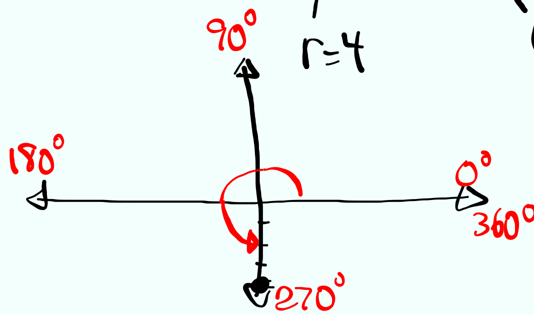
$$= \boxed{-\sqrt{3} - i}$$

Dec 2-11:36 AM

Given $Z = 4 \text{ Cis } 270^\circ$

$= 4(\text{Cos } 270^\circ + i \text{ Sin } 270^\circ)$

$r = 4$ $\quad \theta = 270^\circ$



$= 4(0 + i \cdot (-1))$

$= 0 - 4i = \boxed{-4i}$

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